# Analysis of the backscattering method for single-mode optical fibers

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The theory of the backscattering method, which so far has been known only for multimode fibers, is extended to single-mode fibers. Under certain conditions the result of the present investigation is nearly the same as for multimode fibers although the theory in the latter case is based on a ray optical approach.

## INTRODUCTION

In recent years the backscattering method has become a familiar technique for measuring the attenuation of multimode optical fibers and the location and insertion loss of splices and faults. Its major advantage is that the measurements can be made from one fiber end. A great number of papers have been published presenting experimental results as well as theoretical considerations in this field.[1-3] To our knowledge, however, there exist neither experimental data nor theoretical predictions on the backscattering signal for single-mode fibers. The latter is the object of our investigation. It is important to note that the derivation of formula (12) for the backscattered power appreciably differs from the corresponding analysis in the case of multimode fibers.[3] The reason is that it is inadequate to argue in geometric optical terms such as ray or acceptance angle when dealing with single-mode fibers.

## I. ANALYSIS OF THE SCATTERING PROCESS

A harmonic optical pulse of rectangular envelope is assumed to be coupled into the input end $z = 0$ of a single-mode fiber at time $t = 0$. For simplicity we take the field to be linearly polarized so that we can use scalar equations. At the end of this paper we will drop the latter assumption. Consequently, the electric field of the primary optical pulse propagating in $z$ direction is given by

$$E_i\,(x,y,z,t) = E_0\hat{e}(x,y)\mathrm{rect}\left(\frac{z - v_g t + (l_p/2)}{l_p}\right) \times \cos\beta(z - v_p t)e^{-(1/2)\alpha z}, \quad (1)$$

where $\hat{e}(x,y)$ denotes the unit amplitude field distribution of the fundamental mode, $l_p$ the spatial pulse width, $\alpha$ the power attenuation constant, $\beta$ the phase constant, and $v_{g,p}$ the group and phase velocities, respectively. The function rect is equal to unity if the modulus of its argument is less than 0.5 and zero elsewhere. The scattering is caused by small scale inhomogeneities $\Delta\chi(x,y,z)$ of the local electric susceptibility which act as induced dipole oscillators.[4] The inhomogeneous part $P^*$ of the electric dipole moment per unit volume (or polarization, equivalently) is given by

$$P^*(x,y,z,t) = \epsilon_0\Delta\chi(x,y,z)\,E_i(x,y,z,t). \quad (2)$$

Let us look first of all at the scattering coming from a length element $dz_s$ located at $z = z_s$. Though insignificant from the mathematical point of view, it is important to imagine that $dz_s$ is small as compared with the optical wavelength $\lambda$ since in this case all the dipoles there are excited in phase. This element $dz_s$ is assumed to be subdivided in volume elements $dV_s = dx\,dy\,dz_s$. Calculating the dipole moment $dp$ of each volume element we obtain

$$dp = P^* dV_s = d\hat{p}(x,y,z_s)\mathrm{rect}\left(\frac{z_s - v_g t + (l_p/2)}{l_p}\right) \times \cos\beta(z_s - v_p t), \quad (3)$$

where

$$d\hat{p}(x,y,z_s) = \epsilon_0 E_0\hat{e}(x,y)\Delta\chi(x,y,z_s)e^{-(1/2)\alpha z_s}\,dV_s.$$

In his classical paper (Ref. 5) Goubau treated the problem of the excitation of surface waves by dipoles. Using his result we obtain the differential amplitudes $d^3a_v$ of the fundamental mode excited by each dipole $dp$ and propagating in backward direction:

$$d^3a_v = \frac{\omega\hat{e}(x,y)d\hat{p}(x,y,z_s)}{2\iint[\hat{e}^2(\xi,\eta)/Z]d\xi\,d\eta}. \quad (4)$$

The quantity $\omega = \beta\,v_p$ denotes the light frequency and $Z$ the wave impedance of the medium. Owing to the very small index variations in single-mode fibers (weak guidance), $Z$ can be taken to be constant $Z = \sqrt{\mu/\epsilon}$. The mode fields in single-mode fibers are very nearly Gaussian regardless of the index profile,[6] i.e., $\hat{e}(x,y) = \exp[-(x^2 + y^2)/w_0^2]$. Therefore, we can calculate the amplitude $da_z$ of the fundamental mode excited by the dipoles in $dz_s$:

$$da_z = A(z_s)e^{-(1/2)\alpha z_s}dz_s, \quad (5)$$

where

$$A(z_s) = \frac{E_0}{n\pi w_0^2}\frac{\omega}{c}\iint dx\,dy\,\Delta\chi(x,y,z_s)\exp[-2(x^2 + y^2)/w_0^2].$$

Obviously, the time dependence of the electric field $dE_s$ at the input end due to scattering in the length element $dz_s$ is equal to that of the primary field $E_i$ at $z = 2z_s$. Taking into account the phase difference of $\pi/2$ between incident and backscattered wave[5] and the attenuation of the scattered wavelet on its way back, we obtain

$$dE_s = da_z\,\hat{e}(x,y)e^{-(1/2)\alpha z_s} \times \mathrm{rect}\left(\frac{2z_s - v_g t + (l_p/2)}{l_p}\right)\sin\beta(2z_s - v_p t). \quad (6)$$

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Integration over $z_s$ yields the total backscattered field at the input end

$$E_s(x,y,t) = \hat{e}(x,y)e^{-(1/2)\alpha v_g t} \times \left( \int_{(v_g t - l_p)/2}^{v_g t/2} A(z_s)\sin(2\beta z_s)dz_s \cos\omega t - \int_{(v_g t - l_p)/2}^{v_g t/2} A(z_s)\cos(2\beta z_s)dz_s \sin\omega t \right). \quad (7)$$

Taking the time average $\langle E_s^2 \rangle$ over one optical period $T = 2\pi/\omega$ we find an expression for the backscattered power $P_s$ at the input end of the fiber:

$$P_s = \iint dx\,dy\,\langle E_s^2\rangle/Z = P_0 e^{-\alpha v_g t}\left(\frac{\omega}{c}\right)^2 \frac{G}{n^2(\pi w_0{}^2)^2}, \quad (8)$$

where

$$G = \left| \int_{(v_g t - l_p)/2}^{v_g t/2} dz_s\, e^{j2\beta z_s} \int_{-\infty}^{\infty} \int dx\,dy\,\Delta\chi(x,y,z_s) \times \exp[-2(x^2+y^2)/w_0^2] \right|^2.$$

The quantity $P_0 = (1/2)(E_0^2/Z)(\pi w_0^2/2)$ denotes the power of the incoming optical pulse at $t = 0$.

## II. DERIVATION OF A PRACTICABLE BACKSCATTERING FORMULA

To have an estimate of the backscattered power level we will relate the above quantity $G$ to the attenuation constant $\alpha_s$ caused by scattering. For this purpose let us regard a continuous and homogeneous plane wave with a wave vector $\mathbf{k}_i = (0,0,k)$ incident upon a fixed scattering volume $V$ which is supposed to be of equal length in $z$ direction as the integration volume of the quantity $G$, namely $l_p/2$. The scattered intensity $I_s$ in the far-field at a distance $R$ is given by [see Ref. 4, Eq. (3.3.8)]

$$I_s = I_i \frac{\left(\frac{\omega}{c}\right)^4 (1 - \sin^2\vartheta\cos^2\varphi)}{(4\pi R)^2} \times \left| \int_V dx\,dy\,dz\,\Delta\chi(x,y,z)e^{j\mathbf{Kr}} \right|^2, \quad (9)$$

where $I_i$ is the intensity of the incident plane wave and $\mathbf{K} = \mathbf{k}_i - k\mathbf{e}_R$. The vector $\mathbf{e}_R$ is the unit vector in the direction of the field point while $\varphi$ and $\vartheta$ are its azimuth and polar angle, respectively. We will use the symbol $\tilde{G}(K_x,K_y,K_z)$ for the squared modulus of the volume integral in Eq. (9). It is the three-dimensional power spectrum of $\Delta\chi(x,y,z)$. Since $\Delta\chi$ can be regarded as white spatial noise $\tilde{G}(K_x,K_y,K_z)$ is independent of its arguments and in particular equal to $\tilde{G}(0,0,2\beta)$. By integration over a sphere of radius $R$ we find that the total scattered power $P_t$ is given by $P_t = I_i(\omega/c)^4\,\tilde{G}/6\pi$. The quantity $\tilde{G}(0,0,2\beta)$ is very similar to the quantity $G$ in Eq. (8), except for the Gaussian function. The difference can be removed by replacing the homogeneous incident wave by an inhomogeneous one with

$$I_i = (1/2)(E_0^2/Z)\exp[-2(x^2+y^2)/w_0^2].$$

In this case the total scattered power is

$$P_t = \frac{1}{2}\frac{E_0^2}{Z}\frac{(\omega/c)^4}{6\pi} 2 \left| \int_V dx\,dy\,dz\,\Delta\chi(x,y,z) \times \exp[-2(x^2+y^2)/w_0^2]e^{j2\beta z} \right|^2 = \frac{1}{2}\frac{E_0^2}{Z}\frac{(\omega/c)^4}{3\pi} G \quad (10)$$

This equation can be derived by dividing the volume $V$ into subvolumes $\Delta V = \Delta x \Delta y(l_p/2)$, where $\Delta x$ and $\Delta y$ are small compared with the width $w_0$ of the incident wave, but large compared with the correlation length of $\Delta\chi$, which is of the order of some molecular distances.

It is easy to verify that the attenuation constant $\alpha_s$ caused by scattering can be calculated from

$$\alpha_s = \frac{P_t}{P_0(l_p/2)} = \frac{4(\omega/c)^4}{3\pi^2 w_0^2 l_p} G. \quad (11)$$

This relation is useful because $\alpha_s$, in contrast to $G$, is a quantity of known order of magnitude. Using Eq. (11) to eliminate $G$ in Eq. (8) yields

$$P_s = (1/2)\alpha_s v_g \tau P_0 e^{-\alpha v_g t} S, \quad (12)$$

where

$$S = \frac{3}{2n^2 w_0^2(\omega/c)^2} \simeq \frac{3/2}{\left(\frac{w_0}{a}\right)^2 V^2}\frac{n_1^2 - n_2^2}{n_1^2}.$$

The quantity $\tau = l_p/v_g$ denotes the temporal pulse width, $V$ the normalized frequency of the fiber, $a$ its core radius, and $n_1$ and $n_2$ the refractive indeces on the fiber axis and in the cladding, respectively. The corresponding formula for the backscattered power in multimode fibers[3] is identical to Eq. (12) except for the meaning of the backscattering factor $S$. Under certain assumptions $S$ is equal to $(3/8)(n_1^2 - n_2^2)/n_1^2$ and $(1/4)(n_1^2 - n_2^2)/n_1^2$ for step-index and square-law multimode fibers, respectively.[3]

For step-index single-mode fibers, $S$ can be calculated by using the dependence of $w_0/a$ on the normalized frequency[6]:

$$w_0/a = 0.65 + 1.619\,V^{-3/2} + 2.879\,V^{-6}. \quad (13)$$

From this it follows that the backscattering factor is approximately constant for $1.5 \le V \le 2.4$:

$$0.210\frac{n_1^2 - n_2^2}{n_1^2} \le S \le 0.235\frac{n_1^2 - n_2^2}{n_1^2}. \quad (14)$$

For graded-index single-mode fibers the result is the same within a few percent since they can be treated by using the quantities $a_{\text{eff}}$ and $V_{\text{eff}}$, the effective core radius and the effective normalized frequency.[7,8]

To compare single- and multimode fibers, let us assume for a moment equal index differences and equal coupled powers (the backscattered powers are proportional to these quantities). In this case the backscattered signals are almost the same provided the single-mode fiber is operated at normalized frequencies as indicated above. If we imagine single-mode operation (or if we consider only the fundamental-mode portion of the incident wave and the backscattered wave caused by it) at greater $V$ values, the backscattering factor $S$ is appreciably smaller.

As already mentioned above the assumption of linear polarization can be dropped since the complete form of Eq. (4) contains the scalar product of the dipole moment and the mode field to be excited.[5] For this reason the dipoles excited by one polarization direction do not excite the orthogonal polarization direction. Thus both polarization directions can be treated separately.

## III. CONCLUSIONS

We presented an analysis of the scattering process in single-mode optical fibers and derived a simple formula for the backscattered power that arrives at the input end of the fiber. Nominally, this formula is nearly identical to the corresponding result for multimode fibers. Nevertheless, the backscattered power in single-mode fibers will be considerably smaller. This is due to the fact that the coupled power as well as the index difference will be smaller in practice.

Finally, it should be mentioned that scattering causes crosstalk in optical two-way transmission lines and it limits, for example, the sensitivity of the fiber optical Sagnac interferometer.[9] Therefore, our analysis should also be useful in these cases.

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# Modifying a coherent optical processing system to achieve a measure of redundancy

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**A coherent optical processing system is usually nonredundant because any attempt to secure redundancy by band-limiting the input leads to a corresponding underuse of the Fourier plane. Methods are suggested for putting additional copies of the signal into the unused region. This results in a certain departure from strict coherence but is in no case as extreme as using noncoherent methods.**

A typical optical processing system will have a point monochromatic source in the front focal plane of a condenser, and an image of this will be formed in the back focal plane of the first transform lens, where a filter may usefully be placed. On the face of it, the system is nonredundant since the information passes through it only once. There is a complete analysis of the information present in the input into specific spatial frequencies. On axis we have the zero spatial frequency, which often carries a large fraction of the energy, but gives only the average amplitude of the object and as such does not carry any information about its structure. The information, as commonly understood, is diffracted into the region round the central spot in the Fourier transform plane.

Since each point in the Fourier transform plane corresponds to its own specific spatial frequency, any accident occurring to it (such as dust on the filter) will modify it, and as a consequence it will never be the same again. The output is irretrievably distorted.

It is well known[1–5] that by reason of the finite lateral extent of the source and/or the finite range of wavelengths employed in noncoherent systems, redundancy can occur and dust in the system will produce a loss of contrast but not serious damage to a specific spatial frequency. The degree of redundancy depends on the information supplied being less than the information carrying capacity of the optical pathway: specifically it can be defined as the ratio (informational capacity of the pathway)/(information actually carried). The numerator[6,7] is $E/\lambda^2$, where $E$ is the Etendu of the system.

In other words the input must be band limited and the bandwidth must be less than the band pass of the system in the ratio $1/R$, where $R$ is the redundancy.

This suggests the question: if a band-limited input of the

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